

Density of sea water

The properties of water masses (temperature and salinity) are determined at the sea surface by the interaction with the atmosphere by heat fluxes, evaporation, precipitation, freezing, melting and by the river runoff. When a water mass is not in contact with atmosphere its properties are slowly modified by the mixing with nearby water masses, but the process is sufficiently slow that often is possible to trace back water to the location where it was formed by analyzing its temperature and salinity. Changes in temperature and salinity increase or decrease the density of water, which can trigger vertical motions (convection). Though these changes are very small, they are fundamental because they determine the stability of the water column and the vertical motions in the ocean.

Salinity

Originally salinity was a dimensionless quantity defined as the fraction of dissolved material in parts per thousands (ppt). Note that the range of salinity for most of the ocean's water is from 34.60 to 34.80 ppt, therefore to distinguish salinity values, a measurement accurate to about one part per million is needed. In principle it can be determined by evaporating the water of a sample and weighting the residual. However this is not very practical and it is difficult to get an accurate measurement. Thus the definition of salinity needed some refinement is time

In 1902 the International Council for the Exploration of the Sea published a definition of salinity as the "Total amount of solid materials in grams dissolved in one kilogram of sea water when all the carbonate has been converted to oxide, the bromine and iodine replaced by chlorine and all organic matter completely oxidized." This is not practical it is difficult to get an accurate measurement and to use routinely.

However, the ratios of the various ions in sea water are nearly independent of total salinity and location in the ocean (55.3% Chlorine, 30.8% Sodium, 7.7% Sulfate, 3.7% Magnesium, 1.2% Calcium, 1.1% Potassium), with the exception of very fresh waters close to river estuaries. This allow the use of more practical definitions that are based on the content of chlorine: $S = 0.03 + 1.805Cl$, where *chlorinity Cl* is defined as "the mass of silver required to precipitate completely the halogens in 0.328 523 4kg of the sea-water sample." Therefore, measurement involved a chemical reaction called titration. The definition was corrected in 1964 by UNESCO (which appointed a Joint Panel on Oceanographic Tables and Standards) and became $S = 1.80655 Cl$.

However, estimates using conductivity meters were progressively found more precise and relatively easier to use than the chemical techniques used to measure chlorinity, and consequently the Joint Panel recommended the definition:

$$S = -0.08996 + 28.2929729 R_{15} + 12.80832 R_{15}^2 - 10.67869 R_{15}^3 + 5.98624 R_{15}^4 - 1.32311 R_{15}^5$$

where $R_{15} = C(s,15,0)/C(35,15,0)$ and $C(S, 15, 0)$ is the conductivity of the sea-water sample at 15°C and atmospheric pressure and $C(35, 15, 0)$ is the conductivity of standard "Copenhagen" sea water.

As the use of conductivity became the unique procedure for determining salinity, in 1978 the Joint Panel recommended that salinity be defined using only conductivity, so that all water samples with the same conductivity ratio have the same salinity, independently from their content of chlorine or of total dissolved matter.

$$S = 0.0080 - 0.1692 K_{15}^{1/2} + 25.3851 K_{15} + 14.0941 K_{15}^{3/2} - 7.0261 K_{15}^2 + 2.7081 K_{15}^{5/2}$$

$$K_{15} = C(S, 15, 0) / C(KCl, 15, 0)$$

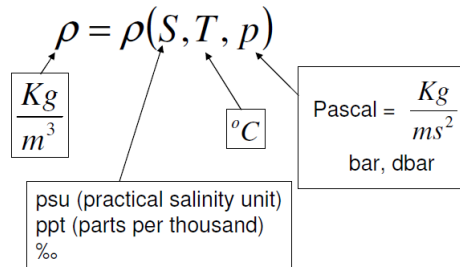
where $C(S, 15, 0)$ is the conductivity of the sea-water sample at a temperature of 15°C and standard atmospheric pressure, and $C(KCl, t, 0)$ is the conductivity of the standard potassium chloride (KCl) solution at a temperature of 15°C and standard atmospheric pressure. The standard KCl solution contains a mass of 32.435 6 grams of KCl in a mass of 1.000 000kg of solution. At temperature T, salinity is corrected adding S_T

$$S_T = [(t - 15) / (1 + 0.0162(t - 15))] (0.0005 - 0.0056 R_t^{1/2} - 0.0066 R_t - 0.0375 R_t^{3/2} + 0.636 R_t^2 - 0.0144 R_t^{5/2})$$

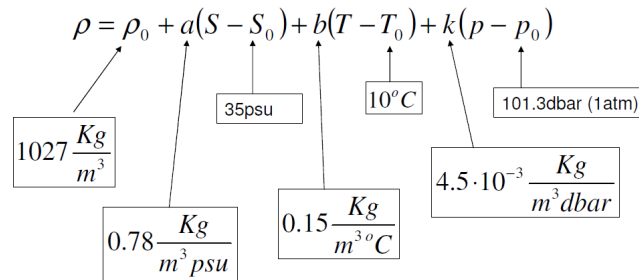
These formulas are valid in the range $2 \leq S \leq 42$, $2^\circ\text{C} \leq T \leq 35^\circ\text{C}$

Density

Density of sea water is a complicated (non-linear) function of temperature, salinity, pressure. The expression is available in most text of oceanography



The linear approximation allows to compute it with a precision up to 0.5‰, which is acceptable in several cases.



It can also be written as $\rho(T, S, p) \approx \rho_o (1 - \alpha_T (T - T_o) + \beta_S (S - S_o) + K (p - p_o))$, where $\alpha_T = -\frac{1}{\rho} \frac{\partial \rho}{\partial T}$ is the thermal expansion coefficient, $\beta_S = \frac{1}{\rho} \frac{\partial \rho}{\partial S}$ is the haline contraction coefficient

for seawater and $K = \frac{1}{\rho} \frac{\partial \rho}{\partial p}$ is the isothermal compressibility coefficient. Typical values for the

reference state are $\rho_o = 10^3 \frac{kg}{m^3}$, $T_o = 283K$, $S_o = 35 \text{ psu}$ and $p_o = 1 \text{ atm}$. Standard values for the Thermal expansion and haline contraction coefficients are $\alpha_T \sim 2 \times 10^{-4} K^{-1}$, $\beta_S \sim 2 \times 10^{-4} \text{ psu}^{-1}$ and $K \sim 4.5 \times 10^{-6} \text{ dbar}^{-1}$.

This linear representation of a non-linear function is only valid for small changes in state variables, and should be applied only when considering small local variations.

The accurate representation of the equation of state that involves the empirical estimate of the secant bulk modulus $K(S, T, p)$, which is expressed as $K = \frac{\rho}{\rho - \rho_0} p$, where K is determined

measuring the density of sea water compressed from the atmospheric pressure p_0 to the value $p_0 + p$. This results in the expression for density

$$\rho(S, T, p) = \frac{\rho(S, T, p_0)}{1 - \frac{p}{K(S, T, p)}}, \text{ where the dependence of density on } T \text{ and } S \text{ at the atmospheric}$$

pressure is given by

$$\begin{aligned} \rho(S, T, p_0) = & 999.842594 + 6.793952 \times 10^{-2} T - 9.095290 \times 10^{-3} T^2 + 1.001685 \times 10^{-4} T^3 \\ & - 1.120083 \times 10^{-6} T^4 + 6.536332 \times 10^{-9} T^5 + 8.24493 \times 10^{-1} S - 4.0899 \times 10^{-3} TS \\ & + 7.6438 \times 10^{-5} T^2 S - 8.2467 \times 10^{-7} T^3 S + 5.3875 \times 10^{-9} T^4 S - 5.72466 \times 10^{-3} S^{3/2} \\ & + 1.0227 \times 10^{-4} TS^{3/2} - 1.6546 \times 10^{-6} T^2 S^{3/2} + 4.8314 \times 10^{-4} S^2 \end{aligned}$$

and

$$\begin{aligned} K(S, T, p) = & 19652.21 + 148.4206T - 2.327105T^2 + 1.360477 \times 10^{-2} T^3 - 5.155288 \times 10^{-5} T^4 \\ & + 3.239908p + 1.43713 \times 10^{-3} Tp + 1.16092 \times 10^{-4} T^2 p - 5.77905 \times 10^{-7} T^3 p \\ & + 8.50935 \times 10^{-5} p^2 - 6.12293 \times 10^{-6} Tp^2 + 5.2787 \times 10^{-8} T^2 p^2 + 54.6746S - 0.603459TS \\ & + 1.09987 \times 10^{-2} T^2 S - 6.1670 \times 10^{-5} T^3 S + 7.944 \times 10^{-2} S^{3/2} + 1.6483 \times 10^{-2} TS^{3/2} \\ & - 5.3009 \times 10^{-4} T^2 S^{3/2} + 2.2838 \times 10^{-3} pS - 1.0981 \times 10^{-5} TpS - 1.6078 \times 10^{-6} T^2 pS \\ & + 1.91075 \times 10^{-4} pS^{3/2} - 9.9348 \times 10^{-7} p^2 S + 2.0816 \times 10^{-8} Tp^2 S + 9.1697 \times 10^{-10} T^2 p^2 S \end{aligned}$$

Table of density values – the following are some values of density given by the International equation of state for seawater (from Pond, S. and Pickard, G. L. *Introductory Dynamical Oceanography*, 2nd ed, Pergamon press, Oxford, UK. 1983, 311.)

S (psu)	$T(^{\circ}C)$	p (bars)	$\rho(S, T, p) \left(\frac{kg}{m^3} \right)$	$K(S, T, p)$ (bars)
0	5	0	999.96675	20337.80375
0	5	1000	1044.12802	23643.52599
0	25	0	997.04796	22100.72106
0	25	1000	1037.90204	25405.09717
35	5	0	1027.67547	22185.93358
35	5	1000	1069.48914	25577.49819
35	25	0	1023.34306	23726.34949
35	25	1000	1062.53817	27108.94504

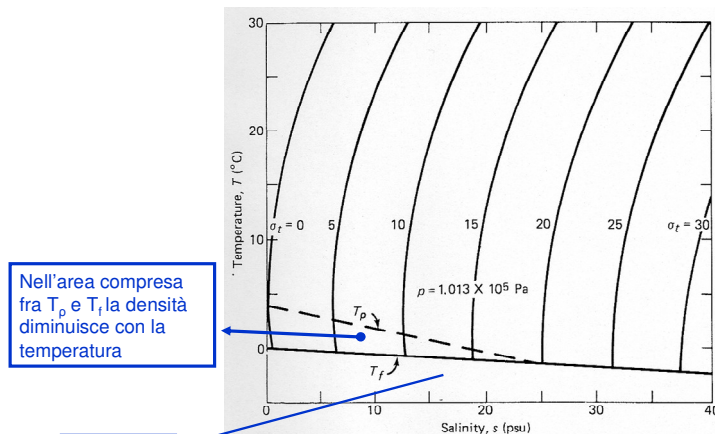


Fig. 4.9 Temperature-salinity-density diagram for seawater over range of normal variations of T and s . This is a cross-plot of Fig. 4.5. [Adapted from Dietrich, G., *Oceanography, An Introductory View* (1968).]

From "Principles of Ocean Physics", J.R. Apel, Academic Press, London and S. Diego.